香港中文大學
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# Analyzing Competitive Influence Maximization Problems with Partial Information

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# Background: Word-of-Mouth

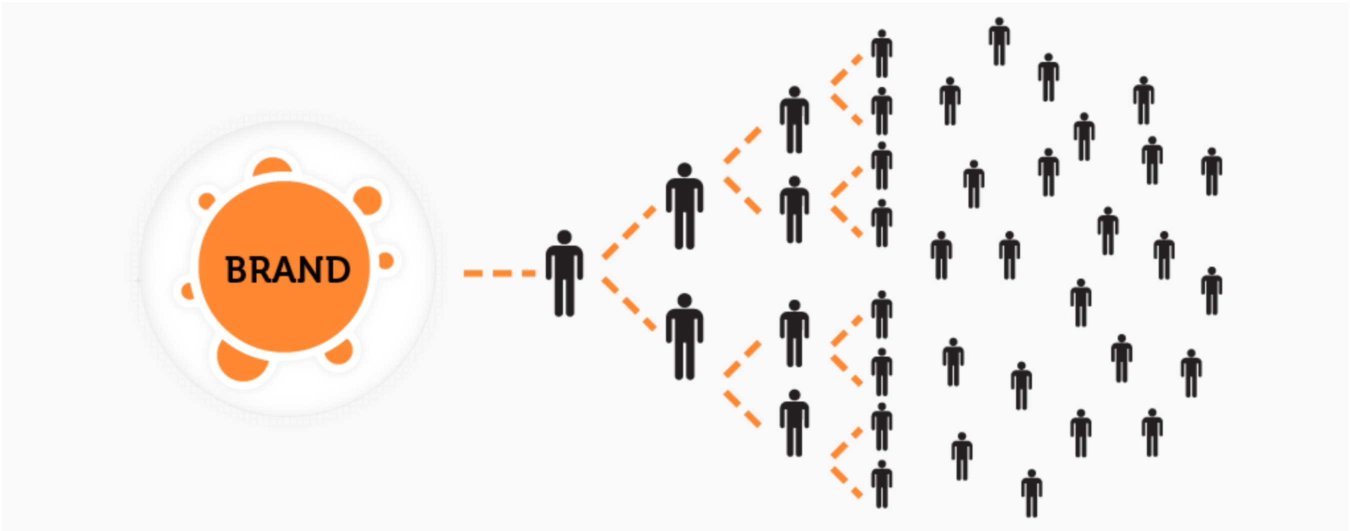
**Foodie** **@Sydney, Australia**

Awesome strawberry watermelon cake with rose scented cream!!

In social interactions, we **influence** each other.

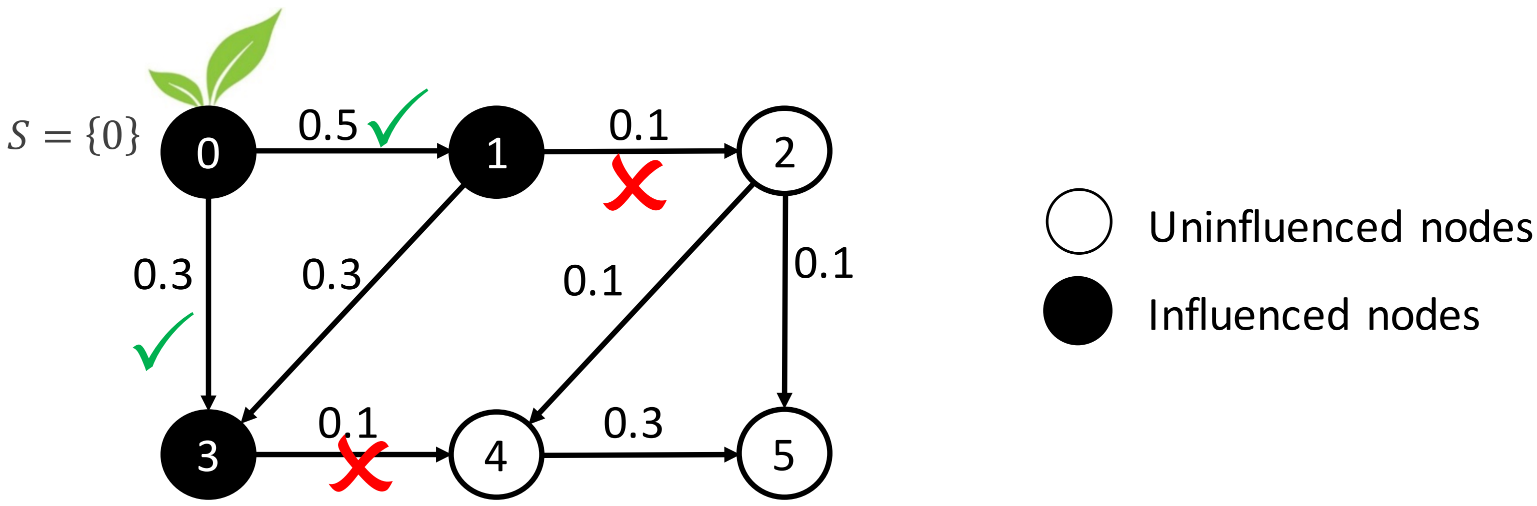
# Background: Viral Marketing

- **Assumption**: the *word-of-mouth* effect
- **Idea**: exploiting the *social influence* for marketing
- **Targeting** "influencers" who are likely to produce the word-of-mouth diffusion



# Background: Classical *Independent Cascade* model

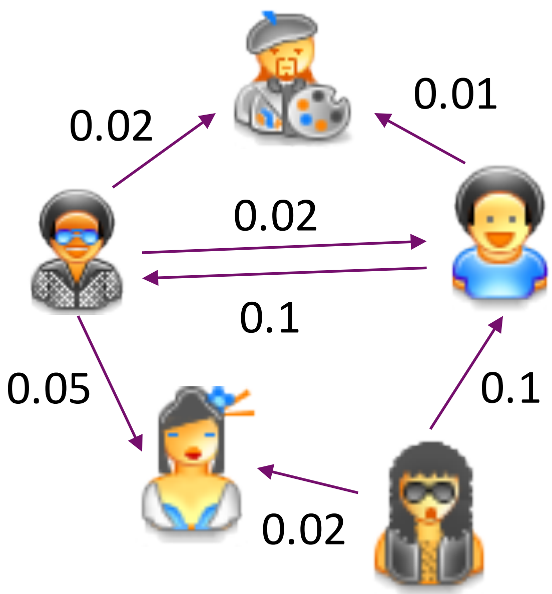
- Single source *Independent Cascade* (IC) model (Kempe et al. KDD'03)
  - Initially, a set of "seed nodes" $S$ are activated.
  - Influenced node $u$ influences its neighbor $v$ with probability $p_{uv}$.
  - **Influence spread $\sigma(S)$**: the expected number of influenced nodes




Kempe, David, Jon Kleinberg, and Éva Tardos. "Maximizing the spread of influence through a social network." *Proceedings of the ninth ACM SIGKDD international conference on Knowledge discovery and data mining*. ACM, 2003.

# Background: Classical *Influence Max. Problem*

**Input:** $G$ and $k$




**Problem** (Influence Maximization)
Select $k$ seed nodes so to maximize the expected spread of influence.

**Output:**
Seed set of size $k$

Under the IC model:

- The IM problem is NP hard. ☹
- Even computing $\sigma(S)$ is #P hard. ☹

# Motivation

- Competition among products


- Partial information: It is not always possible to have full information about viral marketing strategies of the competitor.

# Main Contributions

## General Competitive Independent Cascade Model

- Many specific models proposed previously are its special cases
  - Distance-based model (Carnes et al. ICEC'2007)
  - Wave propagation model (Carnes et al. ICEC'2007)
  - Campaign-Oblivious Independent Cascade model (Budak et al. WWW'11)

## General Competitive Influence Maximization Problem

- Assuming only partial knowledge about competitor's seeding strategy

## General algorithmic framework

- It solves the general problem.
- It works for any specific instances of the general model.

# Model

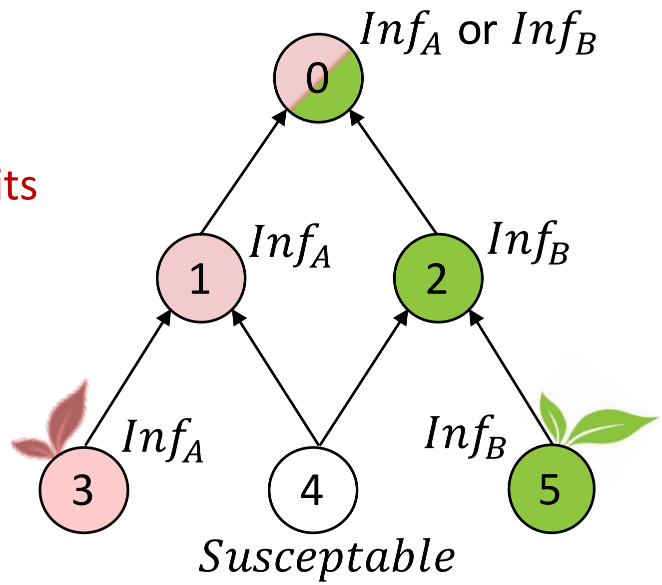
## General Competitive Independent Cascade Model

- **Network** $G = (V, E)$:
  - Every edge $e_{uv}$ is associated with a probability $p_{uv}$.
- **Sources:** two competing sources $A$ and $B$.
- **State of a node:** $Susceptible, Inf_A$ or $Inf_B$
  - "Influenced" cannot change its state.
- **Seeds / initial adopters:** $S_A \subseteq V, S_B \subseteq V$
  - We assume $S_A \cap S_B = \emptyset$.

# Model

## General Competitive Independent Cascade Model

- **Given seeds: $S_A$ and $S_B$**
- **Determine propagation results**
  - Active edges $E_a$: edge $e_{uv}$ is "active" w.p. $p_{uv}$.
  - Node $u$ will be in the same state as that of *one of* its *nearest seeds* in $G = (V, E_a)$.
  - A specific model should specify how the influence propagates in detail.
- **The expected influence**
  - $\sigma(S_B|S_A) = \mathbb{E}_{E_a}[\#\text{ of nodes in state } Inf_B]$
- **Assumption**
  - monotonicity and submodularity of $\sigma(S_B|S_A)$



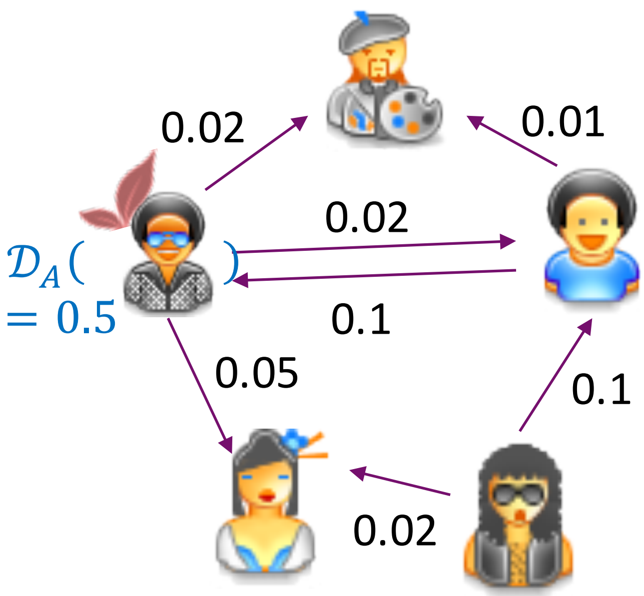

$S_A = \{3\}$ $S_B = \{5\}$

# Problem Definition

Competitive Influence Maximization problem with Partial information (CIMP)

**Input:**

- $G$, $k$, propagation model
- Competitor's seed distribution $\mathcal{D}_A$




**Problem**

Select a set $S_B^*$ of $k$ nodes so that the expected spread of influence of source $B$ under the presence of competitors, $\sigma(S_B^* | \mathcal{D}_A)$, is maximized.

$$= \mathbb{E}_{S_A \sim \mathcal{D}_A} [\sigma(S_B^* | S_A)]$$

Monotone & Submodular

**Output:**

Seed set $S_B$ of size $k$

# Problem Definition

Competitive Influence Maximization problem with Partial information (CIMP)

The CIMP problem is **NP hard.** ☹
Even computing $\sigma(S_B|\mathcal{D}_A)$ is **#P hard.** ☹

# Solution

Two-phase Competitive Influence Maximization (TCIM)

# TCIM: Estimating the Expected Influence

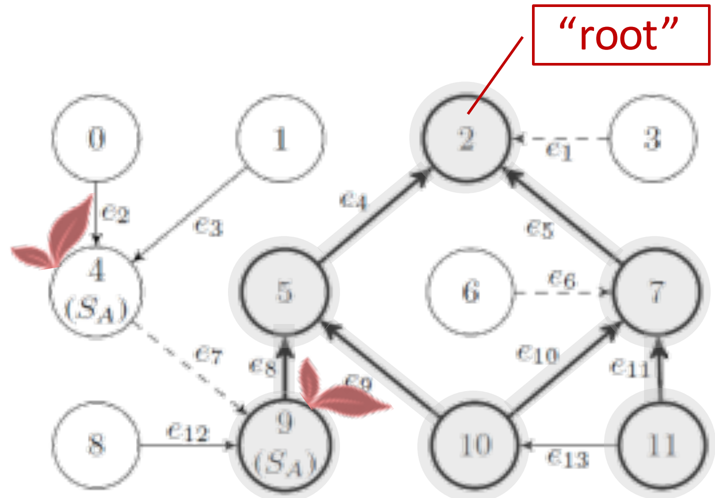
## Random **R**everse **A**ccessible **P**ointed **G**raph (RAPG)

**Input:**

- Random root $r$
- Random seeds $S_A \sim \mathcal{D}_A$
- Random active subgraph $g$

**Output:** $R = (V_R, E_R, S_{R,A})$

- $V_R$: nodes that might influence $r$ in $g$
- $E_R$: all shortest paths from $V_R$ to $r$ in $g$
- $S_{R,A} = S_A \cap V_R$: seeds of source $A$ in $R$.



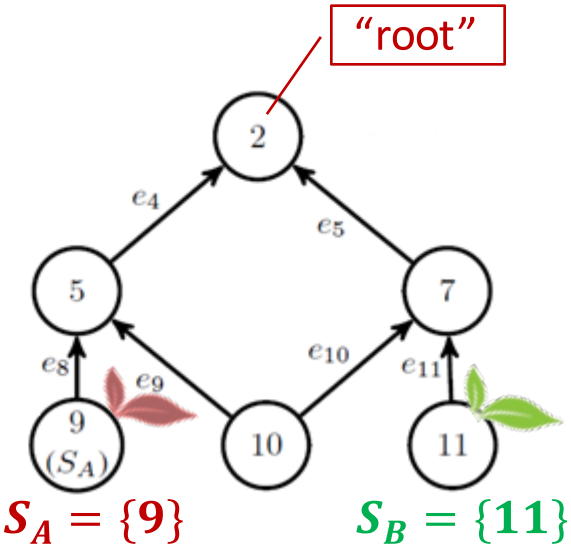

$g = G \backslash \{e_1, e_6, e_7\}, S_A = \{4,9\}$

$R = (V_R, E_R, S_{R,A})$

# TCIM: Estimating the Expected Influence

- **R**everse **A**ccessible **P**ointed **G**raph $R$
- Seed set $S_B$
- Specific competitive propagation model

➡ **"Score" of $S_B$** in $R$:
$\Pr[S_B$ influences the root of $R]$




*Distance-based* Model

"Score" of $S_B$ in $R$: $1/2$

**(Lemma 1)** $n \cdot \mathbb{E}_R[\text{Score}_R(S_B)] = \sigma(S_B|\mathcal{D}_A)$

$\mathbb{E}[$**"Score" of $S_B$** in a random $R]$
$=\Pr[S_B$ influences a random node in $G]$

# TCIM: High Level Ideas

(Lemma 1) $n \cdot \mathbb{E}_R[\text{Score}_R(S_B)] = \sigma(S_B|\mathcal{D}_A)$

Chernoff-Hoeffding Bound

**Estimating Influence**

- Given $\mathcal{D}_A$ and **a sufficiently large number of** random RAPG instances,

$n \cdot \text{ave}[score_R(S_B)] \approx \sigma(S_B|\mathcal{D}_A)$.

**Node Selection** Monotone & Submodular

- We know how to estimate $\sigma(S_B|\mathcal{D}_A)$
- We **greedily add nodes to $S_B$** with the goal of maximizing $\sigma(S_B|\mathcal{D}_A)$

**Phase 1: Parameter estimation**

Estimate and refine the number of RAPG instances we need.

**Phase 2: Node Selection**

1. Generate enough RAPGs
2. Selects seeds for source $B$

# TCIM: Main Results

(Theorem 4) Two-phase Competitive Influence Maximization

**Practical performance guarantee**

- $\sigma(S_B|\mathcal{D}_A) \geq (1 - 1/e - \epsilon) \cdot \sigma\left(S_B^{OPT}\middle|\mathcal{D}_A\right)$, with probability at least $1 - \mathrm{n}^{-\ell}$
- the best approximate ratio one could obtain in polynomial time

**Practical efficiency**

- $O((c(\ell + k)(m + n)\ \log n)/\epsilon^2\ )$
- the value of $c$ is related to the specific GCIC model

# Application: A Special Case of the GCIC model

## **Distance-based Model** (Carnes et al. ICEC'2007)

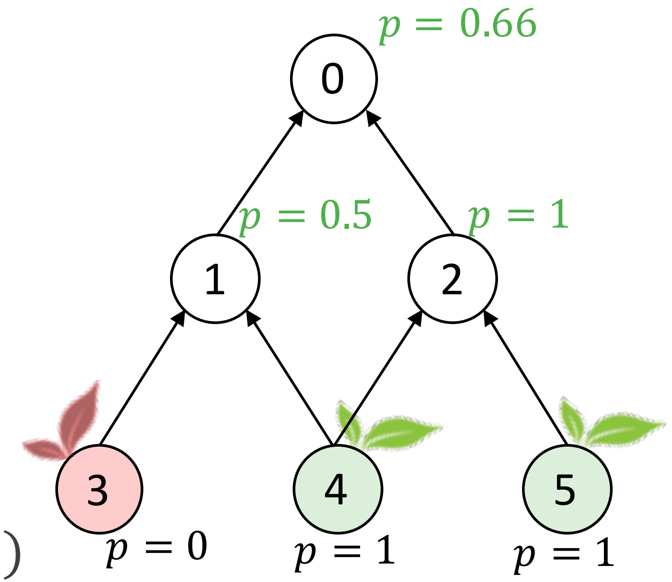
- Given $S_A$, $S_B$ and a set of active edges $E_a$.
- Probability that source B influences node $u$:

$$\frac{\#\text{ of } u'\text{s nearest seeds of source } B}{\#\text{ of } u'\text{s nearest seeds of both sources}}$$

- TCIM Complexity: $O((k(\ell + k)(m+n)\ \log n)/\epsilon^2)$

$$\downarrow$$

$$c = O(k)$$




$S_A = \{3\}, S_B = \{4,5\}$

# Experiments

Comparison among the TCIM framework and previous methods

- Dataset: a *Facebook-like social networks* (1,899 nodes and 20,296 directed edges)
- Baselines:
  - CELF (Leskovec et al. ICDM'07): a greedy method
  - CELF++ (Goyal et al. WWW'11): a greedy method
  - DegreeDiscount (Chen et al. KDD'09): a heuristic method
- Settings:
  - For each edge $e_{uv} \in E$, we set $p_{uv} = 1/d_v^-$ (IC-Weighted Cascade model).
  - We select $50$ nodes using single source influence maximization method for source $A$.

# Comparison among the TCIM framework and previously methods

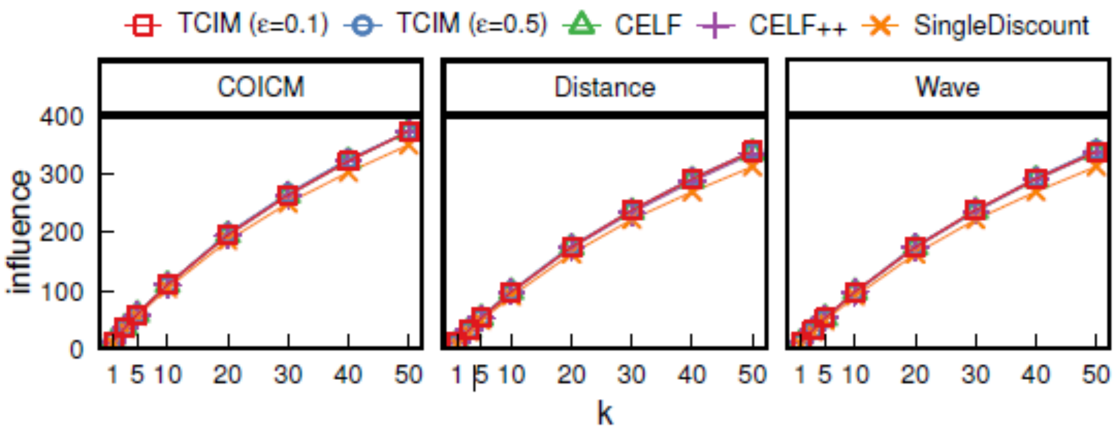



**Figure 3: Results on the *Facebook-like network*: Influence versus $k$ under three propagation models. ($|S_A| = 50$, $\ell = 1$)**

The influence spread of $S_B$ returned by TCIM, CELF and CELF++ are comparable.

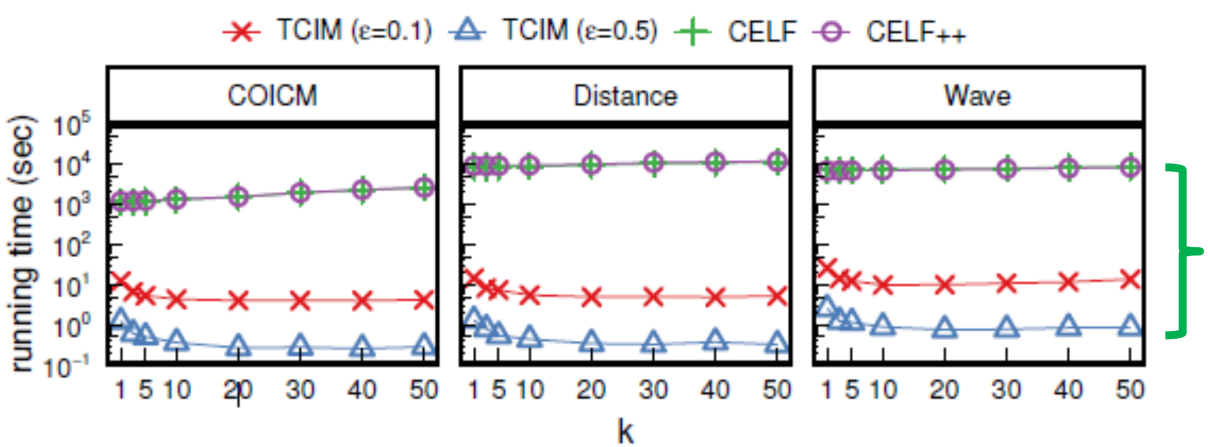



**Figure 4: Results on the *Facebook-like network*: Running time versus $k$ under three propagation models. ($|S_A| = 50$, $\ell = 1$)**

**Up to 4 orders of magnitude speedup**

# Experimental Results

## Results on larger datasets

- The *NetHEPT* collaboration network (15,233 nodes and 58,891 undirected edges)
- The *Epinion* social network (508,837 directed relationships among 75,879 users)

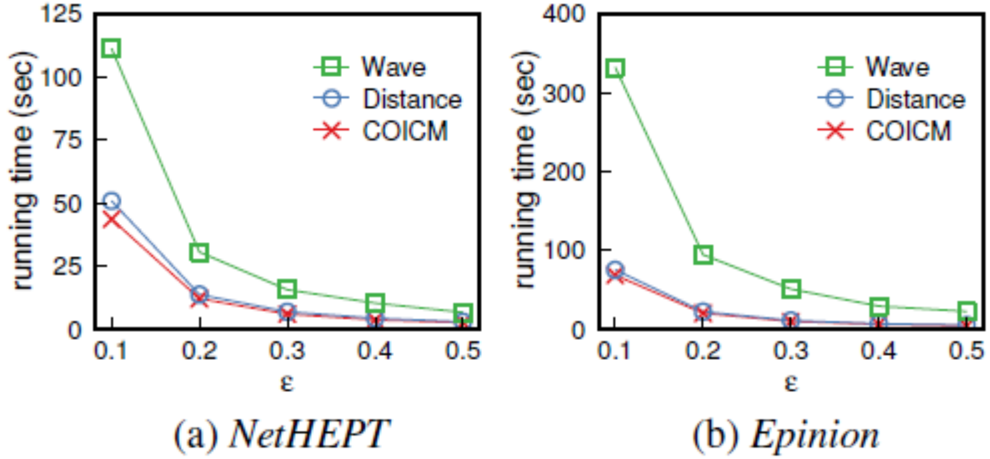



**Figure 7: Results on large datasets: Running time versus $\epsilon$ under three propagation models.** ($|S_A| = 50$, $k = 50$, $\ell = 1$)

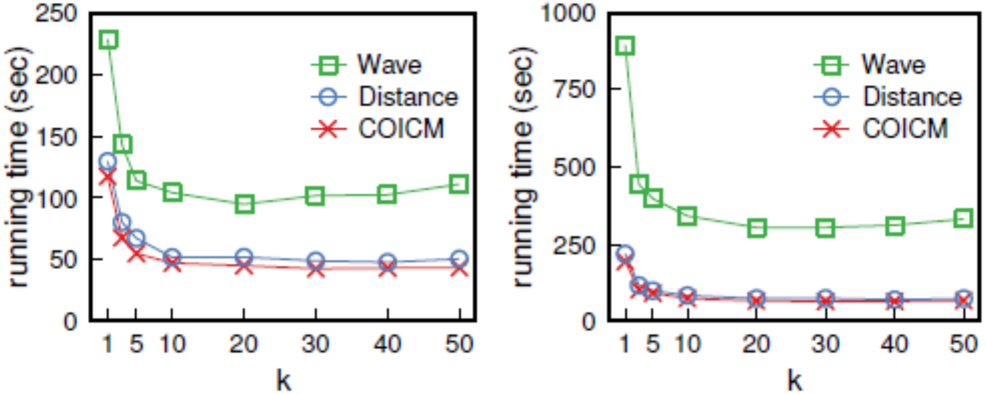
Remarks:

1. When $\epsilon = 0.5$, TCIM finishes within 7 seconds for the *NetHEPT* dataset and finishes within 23 seconds for the *Epinion* dataset.
2. If we do not require a very tight approximation ratio, we could choose a larger $\epsilon$.

# Experimental Results

## Results on larger datasets

- The *NetHEPT* collaboration network (15,233 nodes and 58,891 undirected edges)
- The *Epinion* social network (508,837 directed relationships among 75,879 users)




(a) *NetHEPT* (b) *Epinion*

**Figure 6: Results on large datasets: Running time versus $k$ under three propagation models.** ($|S_A| = 50$, $\epsilon = 0.1$, $\ell = 1$)

Remarks:
1. With the increase of $k$, the running time of TCIM tends to drop first, because the number of RAPG instances needed decreases.
2. TCIM is especially efficient for large $k$.

# Experiments: TCIM with partial information

Competitor's strategy

| dataset | estimated $\mathcal{D}_A/S_A$ | influence given explicit $S_A$ selected by different methods ($\lvert S_A \rvert = 50$) | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | COICM | | | | *Wave propagation model* | | | |
| | | greedy | degree | centrality | average | greedy | degree | centrality | average |
| *NetHEPT* | mixed method | 599.82 | 632.23 | 657.49 | **629.85** | 586.58 | 624.41 | 650.39 | **620.46** |
| | greedy | 658.38 | 515.72 | 519.50 | **564.53** | 644.53 | 525.70 | 515.37 | **561.87** |
| | degree | 400.18 | 702.93 | 622.15 | **575.09** | 372.58 | 693.95 | 613.98 | **560.17** |
| | centrality | 233.14 | 478.74 | 763.43 | **491.77** | 201.72 | 462.66 | 752.97 | **472.45** |
| *Epinion* | mixed method | 2781.71 | 4603.63 | 10683.26 | **6022.87** | 2773.17 | 4494.80 | 10517.00 | **5928.32** |
| | greedy | 4440.93 | 3958.87 | 6372.13 | **4923.98** | 4265.87 | 3813.06 | 6377.30 | **4818.74** |
| | degree | 3130.99 | 5473.33 | 7283.28 | **5295.87** | 2983.56 | 5299.18 | 7258.24 | **5180.33** |
| | centrality | 224.93 | 2809.74 | 12078.70 | **5037.79** | 204.01 | 2721.87 | 12075.78 | **5000.55** |

Table 1: Expected influence of seeds $S_B$ returned by the TCIM framework given the "*mixed method distribution*" (mixed method) as seed distribution for source $A$ or given the guess of explicit seeds of $A$. Seeds "greedy" for source $A$ is the set of nodes selected by single source influence maximization algorithm. The set "degree" for source $A$ (resp. "centrality") denotes the top 50 nodes ranked by (out)degree (resp. closeness centrality). ($k = 50$, $\epsilon = 0.1$, $\ell = 1$)

# Conclusion

- **General problem formulation**
  - *General Competitive Independent Cascade* (GCIC) model
  - *Competitive Influence Maximization problem with Partial information* (CIMP)
- **General *Two-phase Competitive Influence Maximization* (TCIM) framework**
  - It solves the CIMP problem under the GCIC model.
  - With probability at least $1 - n^{-\ell}$, it guarantees a $\boldsymbol{(1 - 1/e - \epsilon)}$**-approximate solution**.
  - It runs in $\boldsymbol{O((c(\ell + k)(n + m) \log n)/\epsilon^2)}$ expected time, where $c$ depends on the specific propagation model.
- We conduct extensive experiments using real datasets. For example,
  - When $S_A$ is given explicitly, we achieve up to **four orders of magnitude speedup** as compared to previous algorithms with the same quality guarantee.

# Thank you!

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